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Supersymmetric AdS_7 backgrounds in half-maximal supergravity and marginal operators of $(1,0)$ SCFTs

Jan Louis and Severin Lüst

Fachbereich Physik der Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

and

Zentrum für Mathematische Physik, Universität Hamburg, Bundesstrasse 55, D-20146 Hamburg, Germany

jan.louis@desy.de, severin.luest@desy.de

ABSTRACT

We determine the supersymmetric AdS_7 backgrounds of seven-dimensional half-maximal gauged supergravities and show that they do not admit any deformations that preserve all 16 supercharges. We compare this result to the conformal manifold of the holographically dual $(1,0)$ superconformal field theories and show that accordingly its representation theory implies that no supersymmetric marginal operators exist.

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1 Introduction

AdS backgrounds of gauged supergravities have been prominently studied in connection with the AdS/CFT correspondence [1]. In particular a large variety of explicit solutions of ten- and eleven-dimensional supergravities of the form $\mathrm{AdS}_d \times Y_{10/11-d}$ have been constructed by now. Generalization of the original $AdS_5 \times S^5$ started in refs. [2,3] and the more recent developments are summarized, for example, in [4]. Refs. [5,6] on the other hand studied AdS_4 backgrounds within four-dimensional ($d = 4$) supergravities without considering any explicit relation with solutions of higher-dimensional supergravities. It was found that the existence of AdS backgrounds imposes specific conditions on the couplings of the supergravity. For $\mathcal{N} = 1$ these conditions are formulated in terms of the Kähler potential and the superpotential. For $\mathcal{N} > 1$ AdS backgrounds can only appear in gauged supergravities and the necessary gaugings are conveniently expressed in terms of the embedding tensor [7,8]. Concretely refs. [5,6] studied $\mathcal{N} = 2$ and $\mathcal{N} = 4$ AdS backgrounds together with their deformations that preserve all supercharges and determined the structure and properties of this moduli space. For $\mathcal{N} = 4$ it is even possible to classify the AdS_4 backgrounds in that the structure of possible gauge groups can be given. In particular a specific subgroup of the R-symmetry group always has to be gauged and has to be unbroken in the AdS background. Furthermore it was shown that no deformations preserving all 16 supercharges exist and one can only have isolated vacua. In [9] the analysis was carried over to AdS_5 backgrounds of

five-dimensional gauged supergravities with 16 supercharges where a similar classification is possible but in this case a moduli space does exist.

In this paper we extend these studies to seven-dimensional supergravities with sixteen supercharges (half-maximal) coupled to an arbitrary number of vector multiplets and determine their AdS_7 backgrounds. Unfortunately, the most general Lagrangian of these supergravities formulated in the embedding tensor formalism is not yet know. The original papers [10, 11] take some of the embedding tensor components into account but not all. This has been partly remedied in [12–14] where all embedding tensor components have been identified. However, so far only certain terms of the full Lagrangian for these additional embedding tensor components have been given. Luckily we can show that in supersymmetric AdS_7 backgrounds only specific embedding tensor components can be non-trivial and for these the Lagrangian is known.

While we work solely within the framework of seven-dimensional gauged supergravity, supersymmetric AdS_7 solutions can be also discussed from the perspective of higher-dimensional supergravity. The only half-maximally supersymmetric solutions in M-theory are of the form $AdS_7 \times S^4/\mathbb{Z}_k$ [15, 16]. There are no supersymmetric AdS_7 solutions in type IIB supergravity, but solutions of massive type IIA supergravity have been explicitly constructed and classified in [17–19]. All these solutions can be truncated consistently to minimal gauged supergravity in seven dimensions [20] and should hence describe a possible higher-dimensional origin for the solutions discussed in this paper.[1]

In our analysis we find that supersymmetric AdS_7 backgrounds require the gauge group G to be of the general form

$$G \;=\; G_0 \times H \subset SO(3,n) \;, \tag{1.1}$$

where n is the number of vector multiplets, H is a compact semi-simple factor while G_0 needs to contain an $SO(3)$ subgroup which has to coincide with the unbroken gauge group in the vacuum. The unbroken $SO(3)$ is the R-symmetry of the supergravity or an admixture of the R-symmetry with an appropriate $SO(3)$ factor associated with the vector multiplets.[2] If we assume the gauge group to be semi-simple, we can further restrict G_0 to be either $SO(3)$, $SO(3,1)$ or $SL(3,\mathbb{R})$. A related result has been obtained in [22, 23] from a different approach.

Furthermore, we study the scalar deformations of the AdS backgrounds which preserve all supercharges and show that they all are Goldstone bosons of the spontaneously broken

[1]In [21] it was however noted that certain solutions of ten-dimensional type IIA string theory (including localized and smeared branes) do not seem to have a description within seven-dimensional gauged supergravity.

[2]Contrary to AdS_4 and AdS_5 backgrounds with 16 supercharges, in $d = 7$ the entire R-symmetry group $SO(3)$ has to be gauged and unbroken.

gauge group G and therefore do not count as physical moduli. Consequently there is no supersymmetric moduli space exactly as for $d = 4$, $\mathcal{N} = 4$ [6].

In the second part of the paper we consider the holographically dual six-dimensional $\mathcal{N} = (1, 0)$ superconformal field theories (SCFT) and their possible exactly marginal deformations. This deformation space is known as the conformal manifold and according to the AdS/CFT correspondence it should coincide with the moduli space of the AdS solutions. In agreement with our previous results we can indeed show that there is no $\mathcal{N} = (1, 0)$ SCFT that can have any supersymmetric marginal deformations and thus no conformal manifold exists. This follows solely from the representation theory of the superconformal algebra in that any possible marginal operator violates the unitarity bounds and therefore is forbidden. Here we essentially follow a similar analysis for $\mathcal{N} = 1$ SCFTs in $d = 4$ performed in [24] and use the $\mathcal{N} = (1, 0)$ representation theory determined in [25–27]. When this manuscript was being completed we learned about ref. [28] which has considerable overlap with the second part of this paper.

This paper is organized as follows. In section 2.1 we briefly review the half-maximal supergravities in $D = 7$. In 2.2 we show that supersymmetric AdS_7 backgrounds imply the gauge group given in (1.1). In 2.3 we show that the resulting scalar potential has flat directions but all of them correspond to Goldstone bosons of a spontaneously broken gauge group in the AdS_7 vacuum. In section 3 we turn to the dual superconformal theories. After discussing some general properties in 3.1 we show in section 3.2 that there are no marginal operators in six-dimensional $\mathcal{N} = (1, 0)$ SCFTs. In Appendix A we review the six-dimensional $\mathcal{N} = (1, 0)$ superconformal algebra, and in Appendix B we discuss the group theoretical restrictions on the level of a Lorentz invariant descendant operator.

2 AdS_7 backgrounds of seven-dimensional half-maximal supergravity

2.1 Preliminaries

In this section we briefly recall the structure of half-maximal gauged supergravities in $d = 7$ following [11–14]. The gravity multiplet has the field content

$$\left(g_{\mu\nu}, \psi^A, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma\right) , \qquad \mu, \nu = 0, \ldots, 6 , \tag{2.1}$$

where $g_{\mu\nu}$ is the metric, ψ^A, $A = 1, 2$ is an $SU(2)_R$-doublet of gravitini, A^i_μ, $i = 1, 2, 3$, is an R-triplet of vectors, χ^A is an $SU(2)_R$-doublet of spin-1/2 fermions, $B_{\mu\nu}$ is an antisymmetric tensor and σ a real scalar. Furthermore there can be n vector multiplets

$$\left(A^r_\mu, \lambda^{rA}, \phi^{ri}\right) , \qquad r = 1, \ldots, n , \tag{2.2}$$

where each consists of one vector A_μ^r, a doublet of spin-1/2 fermions λ^{rA} and a triplet of real scalars ϕ^{ri}. All fermions are symplectic Majorana spinors. Altogether there are $(n+3)$ vector fields, $(2n+2)$ spin-1/2 fermions and $(3n+1)$ real scalars.

The field space $\mathcal{M}$ of the scalars is given by

$$\mathcal{M} \ = \ \mathbb{R}^+ \times \frac{SO(3,n)}{SO(3)\times SO(n)} \ , \tag{2.3}$$

where the $3n$ dimensional coset manifold is spanned by the scalars ϕ^{ri} in the vector multiplet while the $\mathbb{R}^+$ factor corresponds to σ. The coset can be conveniently parametrized by a coset representative[3]

$$L = \left(L_I^i, L_I^r\right) \ , \quad I = 1,\ldots,n+3 \ . \tag{2.4}$$

L is an $SO(3,n)$ matrix and hence satisfies

$$\eta_{IJ} = -L_I^i L_J^i + L_I^r L_J^r \, , \tag{2.5}$$

where $\eta_{IJ} = \mathrm{diag}(-1,-1,-1,+1,\ldots,+1)$ is the canonical $SO(3,n)$ metric. The scalar manifold $\mathcal{M}$ can be described by the metric

$$M_{IJ} = L_I^i L_J^i + L_I^r L_J^r \, . \tag{2.6}$$

The $(n+3)$ vector fields are combined into $A^I = (A^i, A^r)$ and can be rotated into each other by the global symmetry group $SO(3,n)$. A subgroup $G \subset SO(3,n)$ can be made local provided that the structure constants ${f_{IJ}}^K$ of G are completely antisymmetric, i.e. they satisfy the linear constraint

$${f_{IK}}^L \eta_{LJ} + {f_{JK}}^L \eta_{LI} = 0 \ . \tag{2.7}$$

Clearly the dimension of G is restricted by the number of vectors fields to be not larger than $n+3$. As explained in [29] the condition (2.7) restricts the choice of possible (non-compact) gauge groups G strongly. Since η_{IJ} has signature $(3,n)$ any semi-simple G can be either generated by at most three compact or three non-compact generators and the allowed semi-simple gauge groups are cataloged in [12]. In the next section we will determine which of the gauge groups can give rise to AdS vacua.

To construct a gauge invariant action it is convenient to introduce the gauged Maurer-Cartan one-forms

$$P^{ir} = L^{Ir}(\delta_I^K \mathrm{d} + {f_{IJ}}^K A^J) L_K^i \, , \tag{2.8}$$

where L^{Ir} denotes the inverse coset representative. The gauge covariant field strengths are defined by

$$F^I = \mathrm{d}A^I + \frac{1}{2} {f_{JK}}^I A^J \wedge A^K \, . \tag{2.9}$$

[3] In the recent literature on gauged supergravity the coset representatives are often denoted by $\mathcal{V}_I^i$. Here we choose the notation of the original papers [11–14].

Furthermore, for the existence of AdS vacua it turns out to be necessary to dualize the two-form B_2 into a three-form G_3 and to add to the action the topological mass term [12]

$$S_h = 4h \int H_4 \wedge G_3 \ , \tag{2.10}$$

where h is a real constant and $H_4 = \mathrm{d}G_3$ is the four-form field strength.

With these ingredients the total bosonic Lagrangian of gauged $\mathcal{N} = 2$ supergravity reads [11, 12]

$$\begin{aligned} \mathcal{L} \ &= \frac{1}{2} R *1 - \frac{1}{2} e^{\sigma} M_{IJ} F^I \wedge *F^J - \frac{1}{2} e^{-2\sigma} H_4 \wedge *H_4 - \frac{5}{8} \mathrm{d}\sigma \wedge *\mathrm{d}\sigma \\ &\quad - \frac{1}{2} P^{ir} \wedge *P_{ir} - \frac{1}{\sqrt{2}} H_4 \wedge \omega_3 + 4h\, H_4 \wedge G_3 - V *1 \, , \end{aligned} \tag{2.11}$$

where the Chern-Simons three-form ω_3 is given by

$$\omega_3 = \eta_{IJ} F^I \wedge A^J - \frac{1}{6} f_{IJ}{}^K A^I \wedge A^J \wedge A_K \ . \tag{2.12}$$

The potential V takes the form

$$V = \frac{1}{4} e^{-\sigma} \left(C^{ir} C_{ir} - \frac{1}{9} C^2 \right) + 16 h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3}\, h\, e^{\frac{3\sigma}{2}} C \ , \tag{2.13}$$

where we abbreviated

$$\begin{aligned} C &= -\frac{1}{\sqrt{2}}\, f_{ijk} \epsilon^{ijk} \, , \qquad C_{ir} = \frac{1}{\sqrt{2}}\, f_{jkr} \epsilon^{ijk} \, , \\ f_{ijk} &= f_{IJ}{}^K L^I_i L^J_j L_{Kk} \ , \qquad f_{jkr} = f_{IJ}{}^K L^I_j L^J_k L_{Kr} \ . \end{aligned} \tag{2.14}$$

Finally, to find the background solutions that preserve supersymmetry we need the supersymmetry variations of all fermionic fields. They are given by

$$\begin{aligned} \delta\psi_\mu &= \mathrm{D}_\mu \epsilon - \frac{\sqrt{2}}{30}\, e^{-\frac{\sigma}{2}} C \gamma_\mu \epsilon - \frac{4}{5}\, h e^{2\sigma} \gamma_\mu \epsilon + \dots \, , \\ \delta\chi &= \frac{\sqrt{2}}{30}\, e^{-\frac{\sigma}{2}} C \epsilon - \frac{16}{5}\, e^{2\sigma} h \epsilon + \dots \, , \\ \delta\lambda^r &= -\frac{i}{\sqrt{2}}\, e^{-\frac{\sigma}{2}} C^{ir} \sigma^i \epsilon + \dots \, , \end{aligned} \tag{2.15}$$

where we suppressed the R-symmetry index A and the ellipses denote terms which vanish in a maximally symmetric space-time background.

So far we used the supergravity as determined in [11, 12]. However, ref. [13] pointed out that this is not the most general formulation of gauged $\mathcal{N} = 2$ supergravity because

apart from the totally antisymmetric structure constant $f_{[IJK]}$ there can also be another gauge parameter ξ_I which transforms in the vector representation of $SO(3,n)$. Denoting the generators of $SO(3,n)$ by $t_{[IJ]}$ and the generator of the $\mathbb{R}^+$ shift symmetry of σ by t_0, the full embedding tensor is then given by [13]

$$\Theta_I{}^{JK} = f_I{}^{JK} + \delta_I^{[J}\xi^{K]} \ , \qquad \Theta_I{}^0 \ = \xi_I \ , \tag{2.16}$$

and the general covariant derivative reads

$$\mathrm{D} = \mathrm{d} - A^I f_I{}^{JK} t_{JK} - A^I \xi^J t_{IJ} - A^I \xi_I t_0 \ . \tag{2.17}$$

With this information one can determine the Lagrangian of the supergravity. A partial answer has been obtained recently in [14] but the full Lagrangian has not been given yet. Luckily, we will see in the next section that supersymmetric AdS solutions can only occur for $\xi_I = 0$, so that in fact we do not need to use the most general formulation. In order to show this we will need the additional ξ_I dependent terms in the supersymmetry variations given in (2.15). They are of the form [14]

$$\delta\chi \sim e^{-\frac{\sigma}{2}} \xi^i \sigma^i \epsilon + \dots \ , \qquad \delta\lambda^r \sim e^{-\frac{\sigma}{2}} \xi^r \epsilon + \dots \ , \tag{2.18}$$

where $\xi^i = L_I^i \xi^I$, $\xi^r = L_I^r \xi^I$. These variations in turn induce an additional term in the potential given by

$$V_\xi \sim e^{-\sigma}\left(\xi^i\xi^i + \xi^r\xi^r\right) = e^{-\sigma}\xi_I\xi_J M^{IJ} \, . \tag{2.19}$$

2.2 Supersymmetric AdS backgrounds

In this section we derive conditions on the gauge group G such that the theory admits fully supersymmetric AdS vacua. Unbroken supersymmetry implies that the supersymmetry variations of the fermions (2.15) and (2.18) vanish in the AdS background and therefore we need to have

$$\langle C_{ir} \rangle = 0 \, , \qquad \langle C \rangle = \frac{96}{\sqrt{2}}\, h\, e^{\frac{5}{2}\langle\sigma\rangle} \ , \qquad \left\langle \xi^i \right\rangle = \langle \xi^r \rangle = 0 \, . \tag{2.20}$$

As promised we find $\left\langle \xi^I \right\rangle = 0$ which follows from the fact that $(\mathbf{1}, \sigma^i)$ forms a basis of two-dimensional Hermitian matrices and thus the terms given in (2.18) cannot cancel against terms in (2.15). Using the "dressed" structure constants defined in (2.14) the first two conditions in (2.20) read

$$\langle f_{ijk} \rangle = -g\epsilon_{ijk} \, , \qquad \langle f_{ijr} \rangle = 0 \, , \tag{2.21}$$

where the coupling constant g can be chosen arbitrarily and dictates together with h the value of the cosmological constant. Inserted into (2.13), the cosmological constant is

$$\Lambda = \langle V \rangle = -240 h^2 e^{4\langle\sigma\rangle} \ , \tag{2.22}$$

and we indeed see that the background is AdS if and only if a topological mass term with coupling h is included into the action [23, 30]. We also see from (2.20) and (2.21) that the scalar σ from the gravity multiplet has to take the background value

$$\langle\sigma\rangle = \frac{2}{5}\log\left(\frac{g}{16h}\right). \tag{2.23}$$

The conditions (2.21) on the structure constants are very similar to those derived in [6] so that we can essentially follow their analysis for determining the gauge group. The simplest situation occurs when in addition to (2.21) there are no mixed index components of the structure constants, i.e. $f_{ist} = 0$. In this case the gauge group is

$$G = SO(3) \times H \subset SO(3, n)\,, \tag{2.24}$$

where the $SO(3)$ factor is related to the unbroken R-symmetry and $H \subset SO(n)$ has dimension $\dim H \leq n$ and is specified by f_{rst}.[4] Since G is compact it automatically satisfies the condition (2.7) and therefore is an allowed gauge group.

The generic case $f_{ist} \neq 0$ is most conveniently analyzed if we go to a specific basis for the vector multiplet index r where we can split r into $\hat{r}$ and $\tilde{r}$ such that the only non-vanishing components of the structure constants involving an $\tilde{r}$ index are $f_{\tilde{r}\tilde{s}\tilde{t}}$. These components thus correspond to a group $H \subset SO(q)$, $q \leq n$. The remaining components are f_{ijk}, $f_{i\hat{r}\hat{s}}$ and $f_{\hat{r}\hat{s}\hat{t}}$ and they describe a non-compact group $G_0 \subset SO(3, m)$, with $m + q = n$ and $SO(3) \subset G_0$. The total gauge group then is

$$G = G_0 \times H \subset SO(3, n)\,. \tag{2.25}$$

If we furthermore assume that the gauge group is semi-simple, we can list all possible options for G_0 explicitely. From (2.7) we know that G_0 can have either at most three compact or at most three non-compact generators and the only non-compact semi-simple groups satisfying this condition and containing $SO(3)$ as a subgroup are $SO(3, 1)$ and $SL(3, \mathbb{R})$. Therefore G has to be of the form

$$G \;=\; G_0 \times H \;=\; \left\{\begin{matrix} SO(3) \\ SO(3,1) \\ SL(3,\mathbb{R}) \end{matrix}\right\} \times H \;\subset\; SO(3, n)\,, \tag{2.26}$$

where H is an arbitrary semi-simple compact group. This is in agreement with the results from [23], where however the compact factor H was not taken into account for the analysis of AdS vacua.

[4] At the origin of the coset manifold $\mathcal{M}$ the coset representatives are simply delta-functions and the $SO(3)$ factor of the gauge groups corresponds indeed precisely to the $SU(2)$ R-symmetry. However generically L_I^i and L_I^r describe a non-trivial $SO(3, n)$ rotation and the $SO(3)$ factor does not need to coincide directly with the R-symmetry group, but can be embedded into $SO(3, n)$ in a non trivial way.

2.3 Moduli spaces of AdS backgrounds

Let us now compute the moduli space of the AdS backgrounds determined in the previous section. The moduli are the directions in the scalar manifold $\mathcal{M}$ given in (2.3) which are undetermined by the conditions (2.20). Or in other words we are looking for continuous solutions of the variations

$$\delta C_{ir} = 0 \ , \qquad \delta\left(e^{-\frac{5}{2}\sigma}C\right) = 0 \ , \qquad \delta\xi^i = \delta\xi^r = 0 \ . \tag{2.27}$$

The resulting scalar fields are automatically flat directions of the potential (2.13) and thus can be viewed as the scalar degrees of freedom that remain massless in an AdS background.

We proceed along the lines of [6] and parametrize the variations of the coset representatives as

$$\delta L_I^i = \langle L_I^r \rangle \, \delta\phi_{ir} \, , \tag{2.28}$$

where $\delta\phi_{ir}$ are the fluctuations of the $3n$ scalar fields around their background value. Using (2.5) this implies

$$\delta L_I^r = \left\langle L_I^i \right\rangle \delta\phi_{ir} \, , \tag{2.29}$$

while the variations of the inverse coset representatives are given by

$$\delta L_i^I = -\left\langle L_r^I \right\rangle \delta\phi_{ir} \, , \qquad \delta L_r^I = -\left\langle L_i^I \right\rangle \delta\phi_{ir} \, . \tag{2.30}$$

To simplify the notation we will from now on suppress the brackets and assume that all field dependent quantities are evaluated in the background whenever this is appropriate. Since $\xi_I = 0$ it follows directly that $\delta\xi^i = \delta\xi^r = 0$ are satisfied without imposing any conditions on the variations of the scalar fields. From (2.14) and (2.21) we learn

$$\delta f_{ijk} = -3 f_{r[ij}\delta\phi_{k]r} = 0 \, , \tag{2.31}$$

and thus $\delta C = \delta\sigma = 0$. The variation of C_{ir} on the other hand gives the non-trivial condition

$$0 = \delta f_{ijr} = -f_{ijk}\delta\phi_{kr} + 2 f_{rs[i}\delta\phi_{j]s} \, . \tag{2.32}$$

It has been shown in the appendix of [6] that all solutions of this equation are of the form

$$\delta\phi_{ir} = f_{irs}\lambda^s \, , \tag{2.33}$$

where λ^s are arbitrary real parameters. Hence the number of independent moduli is given by the rank of the $(3n \times n)$ matrix $f_{ir\,s}$.[5] Adopting the notation of the previous section we should denote them by $\lambda^{\hat{s}}$ and (2.33) becomes $\delta\phi_{i\hat{r}} = f_{i\hat{r}\hat{s}}\lambda^{\hat{s}}$, $\delta\phi_{i\tilde{r}} = 0$.

[5] The notation should be understood in such a way that the pair of indices ir labels the rows of the matrix $f_{ir\,s}$ while s labels its columns.

The structure constants $f_{i\hat{r}\hat{s}}$ precisely correspond to the non-compact generators of G_0. Since the maximally compact subgroup of G_0 is in every case given by $SO(3)$, we see that the scalar deformations span the coset manifold

$$\mathcal{M}_{\delta\phi} = \frac{G_0}{SO(3)} \,. \tag{2.34}$$

Let us denote by $\tilde{G}$ the maximal subgroup of $SO(3,n)$ that leaves the gauge group G and hence the structure constants invariant. Therefore, acting with $\tilde{G}$ on a solution of (2.21) gives a rotated solution. It is therefore not unexpected that $\mathcal{M}_{\delta\phi}$ is of the form of an orbit of $\tilde{G}$ acting on the scalar manifold $\mathcal{M}$ given in (2.3).

We will now argue that all scalars given in (2.33) are in fact Goldstone bosons eaten by massive vector fields and thus no physical moduli. For this purpose we evaluate the gauged Maurer-Cartan form (2.8) in the AdS background to find

$$P_{ir} = L_i^I \mathrm{d}L_{rI} + f_{irs}A^s \,, \tag{2.35}$$

where $A^s = L_I^s A^I$. This expression appears quadratically in the Lagrangian (2.11) and thus gives a mass term for every vector field A^s in the preimage of the matrix $f_{ir\,s}$. Adopting again our previous notation, (2.35) reads $P_{i\hat{r}} = L_i^I \mathrm{d}L_{\hat{r}I} + f_{i\hat{r}\hat{s}}A^{\hat{s}}$, $P_{i\tilde{r}} = L_i^I \mathrm{d}L_{\tilde{r}I}$ and we see that there is precisely one massive vector field $A^{\hat{s}}$ for every scalar $\lambda^{\hat{s}}$. So no physical massless direction is left and the moduli space can only consist of isolated points.

We can also understand this result directly without analyzing the condition (2.32). The vectors that obtain a mass in the AdS vacuum are in one-to-one correspondence with the non-compact generators of the gauge group G. Therefore the mass term (2.35) breaks the gauge group spontaneously to its maximally compact subgroup, i.e.

$$G = G_0 \times H \to SO(3) \times H \,. \tag{2.36}$$

Breaking G_0 to $SO(3)$ in (2.34) indeed reduces $\mathcal{M}_{\delta\phi}$ to a single point.

3 The conformal manifold of the dual SCFT

In this section we study six-dimensional $\mathcal{N} = (1,0)$ superconformal field theories (SCFTs) which can serve as holographic duals of the AdS backgrounds studied in the previous section. In particular we focus on possible marginal deformations of such SCFTs which preserve all supercharges. We will however show that the representation theory of the $\mathcal{N} = (1,0)$ superconformal algebra forbids any such operators and thus no exactly marginal supersymmetric deformations exist. This is equivalent to the statement that there is no conformal manifold $\mathcal{C}$. The AdS/CFT dictionary relates $\mathcal{C}$ to the moduli space of the dual AdS backgrounds which we studied in the previous section. As on both sides we only find vanishing deformation spaces our results show perfect agreement.

3.1 Preliminaries

Given a SCFT we can deform it by adding conformal operators $\mathcal{O}_i$ to the theory

$$\mathcal{L} \to \mathcal{L} + \lambda^i \mathcal{O}_i \ . \tag{3.1}$$

$\mathcal{L}$ denotes the Lagrangian but this notation is somewhat symbolic as we also consider SCFTs which do not have a Lagrangian description. Operators $\mathcal{O}_i$ that do not break (super-) conformal invariance are called exactly marginal operators. The space spanned by the corresponding exactly marginal couplings λ^i is called the conformal manifold $\mathcal{C}$.

A necessary condition for unbroken conformal invariance is that the λ^i are dimensionless or equivalently that the operators $\mathcal{O}_i$ have conformal dimension $\Delta = 6$, i.e. are marginal operators. This criterion is however not sufficient since higher-order corrections in λ^i can perturb Δ. In the following analysis we only consider marginal operators which do not break the $\mathcal{N} = (1, 0)$ supersymmetry. Thus the $\mathcal{O}_i$ of interest have to be the highest component of a supermultiplet or in other words have to be annihilated by all supercharges. In addition they should be singlets of the R-symmetry group. The superconformal group of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs is the group $OSp(6, 2|2)$ and its representations have been described in detail in [25–27]. Let us briefly recall some of their results which we need for the following discussion.

The bosonic subalgebra of $OSp(6, 2|2)$ is $SO(6, 2) \times SU(2)_R$, where $SO(6, 2)$ is the six-dimensional conformal algebra and $SU(2)_R$ is the R-symmetry. The fermionic part of $OSp(6, 2|2)$ is generated by the supercharges (Q^i_α, S^α_i) where Q^i_α is an R-doublet of chiral spinors with conformal dimension $\Delta = +\frac{1}{2}$, while S^α_i is an R-doublet of antichiral spinors with $\Delta = -\frac{1}{2}$. Here $\alpha = 1, \ldots, 4$, denotes the fundamental representation of $SU(4) = Spin(6)$ and $i = 1, 2$ labels the fundamental representation of the $SU(2)_R$. The representation theory of the superconformal algebra is most conveniently analyzed for the Euclidean theory, where one has the Hermiticity relation $Q^\dagger = S$, so that Q^i_α and S^α_i can be interpreted as ladder operators, raising and lowering the conformal dimension Δ by $\frac{1}{2}$.

As a consequence the unitary irreducible representations of $OSp(6, 2|2)$ decompose into direct sums of representations of the maximally compact subgroup $\mathrm{SO}(2) \times SO(6) \times \mathrm{SU}(2)_R$ of the bosonic subgroup. Each representation can be built from a lowest weight state (conventionally called superconformal primary), which is characterized by the requirement that it is annihilated by all superconformal charges S^α_i. Each primary is labeled by its conformal dimension Δ_0, three half-integer $SO(6)$ weights $h_i = (h_1, h_2, h_3)$ and a half-integer $SU(2)$ weight k.[6] The corresponding supermultiplet is then obtained by successively acting with

[6]It is sometimes convenient to translate the $SO(6)$ weights (h_i) into $SU(4)$ Dynkin labels $[a_1a_2a_3]$ via $a_1 = h_2 - h_3\,,\ a_2 = h_1 + h_2\,,\ a_3 = h_2 + h_3\,.$ This implies in particular that they are not completely arbitrary but that they need to satisfy the constraint $h_1 \geq h_2 \geq |h_3|$. For example $(\frac{1}{2}, \frac{1}{2}, \pm\frac{1}{2})$ denotes the (anti-)chiral

the supercharges Q^i_α on a superconformal primary. A state obtained by the action of l supercharges is called a level-l descendant and it has conformal dimension $\Delta = \Delta_0 + \frac{l}{2}$.

Notice that Δ_0, h_i and k can be used to label the entire supermultiplet. It is however not possible to pick arbitrary combinations of values since unitarity imposes certain constraints. Using the superconformal algebra (see Appendix A) one can compute the norm of the descendant states. Requiring then that all states in a given representation have non-negative norm implies bounds on the conformal dimension Δ_0 of the primary operators which have the generic form

$$\Delta_0 \geq f(h_i, k)\,. \tag{3.2}$$

The function f is explicitly determined in [25, 26] and we recall the results relevant for our analysis in the following section. Representations that saturate the bound (3.2) are short, as in this case some states have vanishing norm and are no longer part of the irreducible representation.

3.2 Classification of marginal operators

After these preliminaries let us go in detail through all possible candidates for supersymmetric marginal operators. As we discussed, they must be part of a unitary representation of the superconformal algebra and therefore are either primary operators or descendant operators that are obtained by acting with l supercharges Q^i_α on a primary operator. However, the primary operators that are invariant under Lorentz-symmetry, R-symmetry and supersymmetry have been shown to be proportional to the identity operator [24]. Therefore we can restrict our further analysis to descendant operators. Among the descendant operators we should also discard those operators where two of the supercharges can be traded for a momentum operator by means of the supersymmetry algebra. These operators add in (3.1) only a total derivative to the Lagrangian and hence do not deform the theory. For the same reason the order of supercharges in a descendant operator does not matter for our analysis.

If we start with a primary operator with $SO(6)$ weights (h_1, h_2, h_3) we can only find Lorentz invariant descendant operators at level

$$l = 2(h_1 + h_2 + h_3) + 4n\,, \tag{3.3}$$

with n being an arbitrary non-negative integer. In Appendix B we give a proof of this statement. Thus the conformal dimension of the primary operator needs to be

$$\Delta_0 = 6 - \frac{l}{2} = 6 - h_1 - h_2 - h_3 - 2n\,. \tag{3.4}$$

spinor representation, while $(1, 0, 0)$ is the $SO(6)$ vector representation.

Moreover, we will use in the following that $k = 0$ is only possible if l is even as descendants with an odd number of supercharges cannot be R-singlets. The general bound from [25, 26] for a unitary representation reads

$$\Delta_0 \geq h_1 + h_2 - h_3 + 4k + 6\,, \tag{3.5}$$

which is not compatible with (3.4), since h_1 and h_2 are necessarily non-negative. Therefore all descendants of primary operators in long representations are excluded.

For special choices of the weights (h_1, h_2, h_3) there exist isolated short representations which we now turn to. The following cases can be distinguished.

a) If $h_1 - h_2 > 0$ and $h_2 = h_3$, there is a short representation with

$$\Delta_0 = h_1 + 4k + 4\,. \tag{3.6}$$

Together with (3.4) the only possible solution is

$$(h_1, h_2, h_3) = (1, 0, 0)\,, \quad k = 0\,, \quad \Delta_0 = 5\,. \tag{3.7}$$

A primary operator with these properties carries no R-symmetry indices and has to be an antisymmetric $SU(4)$-tensor (which is isomorphic to the six-dimensional vector representation of $SO(6)$). Thus the corresponding candidate descendant operator has to take the form

$$\mathcal{O}_2 = \epsilon^{\alpha\beta\gamma\delta} \left\{ Q_{i\alpha}, [Q^i_\beta, U_{[\gamma\delta]}] \right\}\,, \tag{3.8}$$

where $U_{[\gamma\delta]}$ is the associated primary operator with $\Delta_0 = 5$. The norm of this operator can be computed straightforwardly by using the superconformal algebra given in Appendix A with the result $\|\mathcal{O}_2\| \sim \Delta_0 - 5 = 0$. As zero-norm states are not allowed in a unitary theory, the operator $\mathcal{O}_2$ has to vanish.[7]

b) For $h_1 = h_2 = h_3 = h \neq 0$ there are additional short representations if

$$\Delta_0 = 2 + h + 4k\,, \qquad \text{or} \tag{3.9a}$$

$$\Delta_0 = 4 + h + 4k\,. \tag{3.9b}$$

While (3.9b) is not compatible with (3.4), there are two solutions for (3.9a), namely

$$h = \frac{1}{2}\,, \quad k = \frac{1}{2}\,, \quad \Delta_0 = \frac{9}{2}\,, \tag{3.10}$$

and

$$h = 1\,, \quad k = 0\,, \quad \Delta_0 = 3\,. \tag{3.11}$$

[7]Note that this operators is a total derivative for any Δ_0. This is the case because the contraction of the R-symmetry indices is performed with an ϵ-symbol, so $\mathcal{O}_2$ is symmetric under the exchange of the two supercharges and using (A.2a) we see that $\mathcal{O}_2 \sim \left[P^{\alpha\beta}, U_{\alpha\beta}\right]$.

Denoting the primary operator for the first solution (3.10) by U^i_α, it is indeed possible to identify a Lorentz and R-symmetry invariant descendant operator $\mathcal{O}_3$ at level $l = 3$

$$\mathcal{O}_3 = \epsilon^{\alpha\beta\gamma\delta} \left\{Q_{i\alpha}, \left[Q^i_\beta, \{Q_{j\gamma}, U^j_\delta\}\right]\right\} . \tag{3.12}$$

Computing the norm yields $\|\mathcal{O}_3\| \sim \left(\Delta_0 - \frac{9}{2}\right)\left(\Delta_0 + \frac{7}{2}\right)$ and hence vanishes at the critical value $\Delta_0 = 6 - \frac{l}{2} = \frac{9}{2}$. Consequently $\mathcal{O}_3$ itself vanishes and cannot be considered as a possible marginal operator. Notice that it is in principle possible to contract the R-symmetry indices in a different fashion but all such operators differ from $\mathcal{O}_3$ only by a total derivative. Moreover, we have checked that all these other combinations also have vanishing norm.

For the second solution (3.11) the primary operator has the index structure $U_{(\alpha\beta)}$ (with $h = 1$ and $k = 0$) and we can build a Lorentz and R-symmetry invariant descendant operator $\mathcal{O}_6$ at level $l = 6$,

$$\mathcal{O}_6 = \epsilon^{\alpha\beta\gamma\delta}\epsilon^{\epsilon\zeta\eta\theta} \left\{Q_{i\alpha}, \left[Q^i_\epsilon, \left\{Q_{j\beta}, \left[Q^j_\zeta, \left\{Q_{k\gamma}, [Q^k_\eta, U_{(\delta\theta)}]\right\}\right]\right\}\right]\right\} . \tag{3.13}$$

There are also other possibilities to contract the indices within $\mathcal{O}_6$, which would however lead to total derivatives. In any case all these $l = 6$ operators are descendants of the operator $[Q_{i[\alpha}, U_{(\beta]\gamma)}]$, whose norm is $(\Delta_0 - 3)$ and hence vanishes.

c) Finally for $h_1 = h_2 = h_3 = 0$ there are short representations for

$$\Delta_0 = 4k\,, \qquad \Delta_0 = 4k + 2\,, \qquad \Delta_0 = 4k + 4\,. \tag{3.14}$$

Since we have eight distinct supercharges, a descendant operator at level $l > 8$ is always zero by means of (A.2a), so according to (3.3) the only levels at which we should look for suitable candidate operators are $l = 4, 8$.

At level $l = 4$ we need $\Delta_0 = 4$ and there is one operator with $k = 0$,

$$\mathcal{O}_4 = \epsilon^{\alpha\beta\gamma\delta} \left\{Q_{i\alpha}, \left[Q^i_\beta, \left\{Q_{j\gamma}, \left[Q^j_\delta, U\right]\right\}\right]\right\} , \tag{3.15}$$

which has norm $\|\mathcal{O}_4\| \sim \Delta_0(\Delta_0 - 2)$. It does not vanish for $\Delta_0 = 4$, but we find that the norm of $[Q^i_\alpha, \mathcal{O}_4]$ is proportional to $\Delta_0(\Delta_0 - 2)(\Delta_0 + 1)$, so $[Q^i_\alpha, \mathcal{O}_4]$ vanishes only if $\mathcal{O}_4$ itself vanishes. This means that $\mathcal{O}_4$ breaks supersymmetry and thus cannot be a supersymmetric marginal operator. Moreover $\mathcal{O}_4$ is also a total derivative.

The only possibility for non-vanishing k is $k = 1$ as (3.14) implies for $k > 1$ that $\Delta_0 > 4$ while for $k = \frac{1}{2}$ the level l cannot be even. The operator with $k = 1$ reads

$$\mathcal{O}'_4 = \epsilon^{\alpha\beta\gamma\delta} \left\{Q_{i\alpha}, \left[Q^i_\beta, \left\{Q_{j\gamma}, \left[Q_{k\delta}, U^{(jk)}\right]\right\}\right]\right\} . \tag{3.16}$$

We can compute $\|\mathcal{O}'_4\| \sim (\Delta_0 - 4)(\Delta_0 + 6)(\Delta_0 + 8)$, and thus this operator is ruled out as well. Clearly it is again also a total derivative.

At level $l = 8$ we need $\Delta_0 = 2$. Using the same argument as above there is no operator with $k \neq 0$. Hence a Lorentz invariant level $l = 8$ operator is (up to total derivatives) always a descendant of the $l = 2$ operator

$$\mathcal{O}^{ij}_{\alpha\beta} = \left\{ Q^i_{[\alpha}, \left[Q^j_{\beta]}, U \right] \right\} . \tag{3.17}$$

If we antisymmetrize also in the R-symmetry indices i and j, we find $\left\| \mathcal{O}^{[ij]}_{\alpha\beta} \right\| \sim \Delta_0$, but this operator is symmetric under the exchange of the two supercharges and we end up with a total derivative. On the other hand we find for the symmetric component that $\left\| \mathcal{O}^{(ij)}_{\alpha\beta} \right\| \sim \Delta_0(\Delta_0 - 2)$, so it vanishes at the dimension we are interested in. Let us show for the sake of completeness that also all the level $l = 8$ descendants of $\mathcal{O}^{[ij]}_{\alpha\beta}$ have vanishing or negative norm at $\Delta_0 = 2$. They are in turn descendants of the $l = 4$ operator

$$\mathcal{O}^{ij} = \epsilon^{\alpha\beta\gamma\delta} \left\{ Q^i_\alpha, \left[Q^j_\beta, \epsilon_{kl} \mathcal{O}^{kl}_{\gamma\delta} \right] \right\} = \epsilon^{\alpha\beta\gamma\delta} \left\{ Q^i_\alpha, \left[Q^j_\beta, \left\{ Q_{k\gamma}, \left[Q^k_\delta, U \right] \right\} \right] \right\} . \tag{3.18}$$

While the antisymmetric part $\mathcal{O}^{[ij]}$ of this operator is nothing else than $\mathcal{O}_4$ from (3.15) with norm $\Delta_0(\Delta_0 - 2)$, the symmetric part $\mathcal{O}^{(ij)}$ has norm $\Delta_0(\Delta_0 - 2)(\Delta_0 - 4)$, and so both operators have vanishing norm for $\Delta_0 = 2$.

To conclude we have thus shown that all candidates for marginal operators either have zero norm or are not supersymmetric. Notice that most of the operators are also total derivatives but we did not have to use this fact in our argument. Let us close with the observation that the above analysis can be easily extended to relevant operators with conformal dimension $\Delta < 6$. In this case the dimension of the primary operator needs to satisfy

$$\Delta_0 = \Delta - \frac{l}{2} < 6 - h_1 - h_2 - h_3 - 2n , \qquad n \in \mathbb{N} , \tag{3.19}$$

which is clearly also not compatible with the general bound (3.5). Moreover for generic $\Delta < 6$ all isolated short representations are ruled out as well. Only for $\Delta = 4$ the operators from c) with $k = 0$ remain possible candidate operators, but we have shown that their norms are negative at the appropriate dimensions.

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Appendix

A The $\mathcal{N} = (1, 0)$ superconformal algebra

In this appendix we review the relevant (anti-) commutator relations of the six-dimensional $\mathcal{N} = (1, 0)$ superconformal algebra $OSp(6, 2|2)$. The conformal group $SO(6, 2)$ is generated by the Lorentz generators $M_{\mu\nu}$, the momenta P_μ, the special conformal generators K_μ and the dilatation operator D. The generators of the R-symmetry group $SU(2)$ are denoted by R_i^j, where $i, j = 1, 2$. In addition there are the supercharges Q_α^i, with $\alpha = 1, \ldots, 4$, and the superconformal charges S_i^α, which together span the fermionic part of $OSp(6, 2|2)$.

It is convenient to use the local isomorphism $SO(6) \cong SU(4)$ to label also the generators of the conformal group in an $SU(4)$ covariant way, i.e. the Lorentz generators become M_β^α (with $M_\alpha^\alpha = 0$) and the momenta and special conformal generators become $P_{[\alpha\beta]}$ and $K_{[\alpha\beta]}$ respectively.

Since the commutation relations involving only bosonic operators are not relevant for our analysis and can be found for example in [25], we only give the fermionic (anti-)commutators. These are

$$\begin{aligned}
\left[D, Q_\alpha^i\right] &= -\tfrac{i}{2} Q_\alpha^i\,, \\
\left[D, S_i^\alpha\right] &= \tfrac{i}{2} S_i^\alpha\,, \\
\left[M_\beta^\alpha, Q_\gamma^i\right] &= -i\left(\delta_\gamma^\alpha Q_\beta^i - \tfrac{1}{4}\delta_\beta^\alpha Q_\gamma^i\right)\,, \\
\left[M_\beta^\alpha, S_i^\gamma\right] &= i\left(\delta_\beta^\gamma S_i^\alpha - \tfrac{1}{4}\delta_\beta^\alpha S_i^\gamma\right)\,, \\
\left[R_j^i, Q_\alpha^k\right] &= -i\left(\delta_j^k Q_\alpha^i - \tfrac{1}{2}\delta_j^i Q_\alpha^k\right)\,, \\
\left[R_j^i, S_k^\alpha\right] &= i\left(\delta_k^i S_j^\alpha - \tfrac{1}{2}\delta_j^i S_k^\alpha\right)\,,
\end{aligned} \tag{A.1}$$

and

$$\left\{Q_\alpha^i, Q_\beta^j\right\} = \epsilon^{ij} P_{\alpha\beta}\,, \tag{A.2a}$$

$$\left\{S_i^\alpha, S_j^\beta\right\} = \epsilon_{ij} K^{\alpha\beta}\,, \tag{A.2b}$$

$$\left\{S_i^\alpha, Q_\beta^j\right\} = i\left(2\delta_i^j M_\beta^\alpha - 4\delta_\beta^\alpha R_i^j + \delta_\beta^\alpha \delta_i^j D\right)\,. \tag{A.2c}$$

B Level of Lorentz-invariant descendant states

In this appendix we discuss at which levels it is possible to find a Lorentz-invariant descendant state, starting from a superconformal primary with given $SO(6)$ weights (h_1, h_2, h_3). Let us denote the minimal level at which this is possible by N and notice that we will then also find Lorentz invariant states at the levels $l = N + 4m$, $m \in \mathbb{N}$.

The problem is conveniently analyzed in the language of $SU(4)$ Young tableau, since here N corresponds to the number of boxes that need to be added to the diagram to fill up every of its columns to the maximal length four. More generally if we switch to an arbitrary $SU(n)$ Young tableau and call the length of its i^{th} row r_i and the length of its i^{th} column l_i, N is given by

$$N = \sum_{i=1}^{r_1} (n - l_i) \ , \tag{B.1}$$

where the sum runs over all columns. If we use the fact that the lengths of the columns and rows are related via

$$l_i = p \qquad \text{for} \quad r_{p+1} < i \leq r_p \ , \quad p = 1, \ldots, n-1 \ , \tag{B.2}$$

and that the Dynkin labels a_i can by read off from the tableau by

$$a_i = r_i - r_{i+1} \, , \tag{B.3}$$

where $r_n \equiv 0$, we find

$$N = \sum_{i=1}^{n-1} (n - i) \, a_i \, . \tag{B.4}$$

Going back to the relevant case $n = 4$ and using that $a_1 = h_2 - h_3$, $a_2 = h_1 + h_2$, $a_3 = h_2 + h_3$, the result reduces to

$$N = 2 \, (h_1 + h_2 + h_3) \ . \tag{B.5}$$

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